REVIEW

of the official reviewer of the dissertation
"Countable Models of Small Dependent Theories" by Zambarnaya Tatyana,
submitted for the degree of Doctor of Philosophy (PhD)
in the specialty 6D060100 — Mathematics

1. The relevance of the research topic and its relationship with general scientific and national programs. The thesis work by T. Zambarnaya is devoted to model-theoretical questions relating to the structure and spectrum of models of elementary theories, namely, to the study of the structure and number of countable models of small dependent theories.

The active study of the structure and spectrum of models of elementary theories began in the 1960s at the works by R. Vaught, M. Morley, then continued in the works by J. Baldwin, A. Lachlan, S. Shelah, E.A. Palyutin, O.V. Belegradek, B.I. Zilber, R. Woodrow, D. Lascar, B. Poizat, A. Pillay, A. Tsuboi, S.S. Starchenko, S. Buechler, E. Hrushovski, S.V. Sudoplatov, P. Tanovic. B. Kim and others. Important structural results related to the number of models of the theory, along with the results of the author of the dissertation, were obtained by Kazakhstan mathematicians B.S. Baizhanov, E.R. Baisalov, M.I. Bekenov, T.G. Mustafin, B.I. Omarov, K.Zh. Kudaibergenov, B.Sh. Kulpeshov, T.A. Nurmagambetov, M.G. Peretyatkin M.M. Yerimbetov et al. To this day, the study of models of the theory and the description of their spectrum remain highly of current interest and attracts a number of well-known specialists in the field of the theory of models and related fields. One of the most intriguing is the open problem of Vaught on the existence of a theory with uncountably many but less than continuum many countable models. This problem is a significant catalyst for the development of structural theory. In the direction of a partial solution of this problem, a number of structural results have been obtained in this dissertation

The results presented in the thesis are aimed at developing the theory of models for dependent theories. The main scientific studies were carried out in the framework of the basic research programs conducted by Institute of Mathematics and Mathematical Modeling of the Ministry of Education and Science of the Republic of Kazakhstan and Al-Farabi Kazakh National University. The results of the work are included in the annual scientific reports on basic research programs.

2. Scientific results and their validity. Note the main results of the thesis by T. Zambarnaya:

– It is proved that for any countable theory of an expansion of linear order from the existence of a finite set and a non-principal extremely trivial type over this set it follows that this theory has maximum many countable non-isomorphic models;

– It is shown that if in a countable complete theory there is a formula that defines a partial order on elements or on tuples of elements with the condition of existence for each natural number of a discrete chain whose length is not less than this number, then this theory has continuum many countable models;

– It is proved that for any countable theory of an expansion of linear order if there exists a formula quasi-successor on some non-principal 1-type then this theory has maximum many countable non-isomorphic models;

– The Vaught conjecture is confirmed for the class of weakly o-minimal theories of convexity rank 1.

The thesis consists of three introductory sections, including a historical overview and preliminaries, six main sections, conclusion and the bibliography. In the introduction, the main used notions and notations used are given, the relevance of the topic is substantiated, a historical overview is given, and the main objectives of the research are outlined.

In the first main section (Section 3), it is proved that under the assumption that there are two countable families of types with the condition of their independent realizability and omission in the models of this theory and more than ω finite diagrams, there is a maximum number of countable models of this theory (Theorem 3.1).

Section 4 examines the weak and almost orthogonality of types. Sufficient conditions are proved for an infinite number of countable models in the presence of countable families of orthogonal non-principal types (Theorems 4.1 and 4.2).

In Section 5, the author studies the conditions for the triviality of types both in the general case and on linear orders. A sufficient condition is proved for an infinite number of countable models of a theory enriched with a finite number of constants, in the presence of an eventually extremely trivial nonisolated type over the set of these constants (Theorem 5.2.1). It has been established that for any countable theory of some linear ordering from the existence of a finite set and a non-principal extremely trivial type over this set, it follows that this theory has the maximum number of countable non-isomorphic models (Theorem 5.2.3).

Section 6 shows that if there is a formula in a countable complete theory that determines a partial order on elements or on tuples of elements with a condition of existence for each natural number of a discrete chain whose length is not less than this number, then this theory has continuum many countable models (Theorem 6.1).

In Section 7, it is proved that for any countable theory of an expansion of linear order if there exists a formula quasi-successor on some non-principal 1-type then this theory has maximum many countable non-isomorphic models (Theorem 7.1).

Section 8 confirms the Vaught conjecture for the class of weakly o-minimal minimal theories of convexity rank 1.

Thus, the thesis justified and resolved important structural questions about the structure and number of countable models of dependent theories, including the confirmation of the well-known Vaught conjecture for the class of weakly o-minimal theories of convexity rank 1.

3. The degree of validity and reliability of each scientific result, conclusions and conclusions of the applicant, formulated in the dissertation. The scientific results presented in the thesis correspond to the passport of the specialty 6D060100 - Mathematics and are a significant contribution to the development of the theory of countable models of elementary theories, including the structure of these models and the description of their number up to isomorphism.

The thesis has an internal unity: all the main results relate to countable models of elementary theories.

The reliability of all the results obtained in the dissertation is confirmed by rigorous mathematical proofs and necessary calculations, as well as their testing at scientific conferences and seminars.

4. The degree of novelty of each scientific result, the conclusion of the applicant, formulated in the thesis. All the main results of the thesis are new and interesting, timely published in leading mathematical journals.

5. Practical and theoretical significance of scientific results. All the main results of the thesis are of theoretical importance and can be used for special courses and further research in the field of the theory of models in the leading scientific centers of Kazakhstan, Russia and other countries.

6. Comments, suggestions for the dissertation. We note some comments on the text of the dissertation:

1) on p. 14 the article [49] is mentioned in which it is a question of the distribution of countable models of the theory with respect to semi-isolation and Rudin-Keisler preorders; this article is rather of a review nature and cannot be considered as a primary source on the indicated subject;

2) on p. 72 the sign of multiplication is mistakenly replaced by the sign ó;

3) on p. 73 it should be written that the number of countable models of Ehrenfeucht theories is not between 2 and ω, but between 3 and ω;

4) on p. 77 it is necessary to write that the paper [48] was published in Siberian Electronic Mathematical Reports.

These shortcomings are easily removable and do not have a significant impact on the general understanding of the text. In general, the design of the dissertation is good, the proofs, many of which are very non-trivial, are detailed and thorough.

7. Compliance with the content of the thesis in the framework of the requirements of the Rules for the award of scientific degrees. I believe that the thesis by Tatyana Zambarnaya "Countable Models of Small Dependent Theories" fully satisfies the requirements for PhD dissertations in the specialty 6D060100 — Mathematics, meets the requirements of the "Rules for the award of scientific degrees" of the Committee for Control of Education and Science of the MES RK, and its author - Tatyana Zambarnaya, no doubt, deserves the award of the degree of Doctor of Philosophy (PhD).

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